



**national  
accelerator  
laboratory**

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**Section**

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**Subject** ELECTRIC FIELD REQUIRED FOR DE-NEUTRALIZING THE  
STACKED BEAM IN THE NAL STORAGE RINGS

## 1. Uniform Charge Distribution

To obtain a rough estimate, we assume a beam of circular cross-section, radius  $a$ , and uniform charge distribution  $\rho$ . The maximum electric field, which is at the surface of the beam, is

$$E(a) = \frac{\rho a}{2 \epsilon_0}.$$

If  $N$  is the total number of particles in a storage ring and  $R$  the mean radius of the ring

$$a \rho = \frac{Ne}{2\pi^2 Ra},$$

and hence

$$E(a) = \frac{Ne}{4\pi^2 \epsilon_0 Ra}.$$

Putting  $N = 10^{15}$ ,  $R = 333$  m,  $a = 5 \times 10^{-3}$  m,  
we obtain  $E(a) = 2.76 \times 10^5$  V m<sup>-1</sup>.

With a vertical aperture of 0.025 m, the voltage between a single clearing electrode and the vacuum chamber would be about 7 kilovolts.

## 2. Gaussian Charge Distribution

The potential arising from a rotationally symmetrical charge distribution, gaussian in radius is

$$V(r) = - \frac{\rho_0 a^2}{4 \epsilon_0} \int_0^\infty \left[ \frac{1 - e^{\left(\frac{-r^2}{a^2 + t}\right)}}{(a^2 + t)} \right] dt,$$

where the charge distribution is defined by

$$\rho(r) = \rho_0 e^{-\frac{r^2}{a^2}}.$$

The electric field is

$$E(r) = - \frac{\partial V}{\partial r} = \frac{\rho_0 a^2 r}{2 \epsilon_0} \int_0^\infty \frac{e^{-\frac{r^2}{a^2+t}}}{(a^2+t)^2} dt,$$

which can be transformed by the substitution

$$u = \frac{r^2}{(a^2+t)},$$

into

$$E(r) = \frac{\rho_0 a^2}{2 \epsilon_0 r} \int_{\frac{r^2}{a^2}}^0 e^{-u} du = \frac{\rho_0 a^2}{2 \epsilon_0 r} \left[ 1 - e^{-\left(\frac{r^2}{a^2}\right)} \right].$$

A maximum of  $E(r)$  is given by  $\frac{\partial E}{\partial r} = 0$ ,

which leads to

$$e^{-\left(\frac{r^2}{a^2}\right)} \left[ \frac{2r^2}{a^2} + 1 \right] - 1 = 0.$$

The numerical solution of interest is

$$\frac{r}{a} \approx 1.12,$$

leading to

$$\left[ E(r) \right]_{\max} = \frac{\rho_o a}{2\epsilon_o} \times 0.64.$$

The linear charge density  $\lambda$  is

$$\lambda = \int_0^{\infty} \rho(r) 2\pi r dr = \rho_o \pi a^2,$$

and since  $\lambda = \frac{Ne}{2\pi r}$ ,

$$\left[ E(r) \right]_{\max} = 0.64 \frac{Ne}{4\pi^2 \epsilon_o R a}.$$

If we define the "measurable" beam as that contained inside a radius  $\sqrt{2} a$  (which contains 86.5% of the charge), we can write

$$\left[ E(r) \right]_{\max} = 0.905 \frac{Ne}{4\pi^2 \epsilon_o R (\sqrt{2} a)},$$

which shows that, with reasonable definitions of beam dimensions, the first assumption of uniform distribution gives a peak field about 10% greater than a gaussian. Practical distributions are likely to lie somewhere between these limits.